Some ideas for a finite geometry of interaction model of second-order MLL

Nguyễn Lê Thành Dũng (LIPN, Université Paris 13)

nltd@nguyentito.eu

In a recent paper[1], we showed that in second-order Multiplicative Linear Logic (MLL2), the cut-elimination between a proof net $\pi : A$ and a "test" $\rho : A \multimap B$ (B quantifier-free) can be performed as a sort of "dialogue": the two proofs exchange positions of axiom links whose endpoints are atoms of A. The observational equivalence class of π can therefore be determined by a "strategy" of bounded size, proving that the observational quotient of MLL2 is finite at each type. However, the ad-hoc notion of strategy used does not compose.

An attempt to build an actual denotational semantics based on these ideas is currently underway. A first idea is to use as "strategies" *stable functions* $L_\exists \to L_\forall$ where $L_\exists$ (resp. $L_\forall$) is a coherence space whose cliques correspond to (partial) axiom linkings over the existential (resp. universal) atoms of A. Then, from the strategies of π and ρ, the quantifier-free observation can be characterized by a "feedback equation" à la Geometry of Interaction (GoI).

Next, we consider the compact closed category $\mathcal{C} = \mathsf{Int}(\mathsf{CohS})$ i.e. the Joyal–Street-Verity Int construction applied to the category CohS of coherence spaces and stable functions. It is a "wave-style" GoI model of propositional MLL, and the goal is to interpret quantifiers so that the semantics of closed types fit with the specifications of the previous paragraph. This would give a finite semantics for MLL2.

It seems that this model (which, at the time of writing, still depends on unproven lemmas!) exhibits a phenomenon of *inversion of polarities*: the universal quantifier behaves like a positive connective. Indeed, a universal proof plays axiom links, while an existential proof reacts to them (to compare with Hughes's hypergames, it is as if, after a copycat link is first set up by a polymorphic program, memorizing it for the rest of the interaction were the responsibility of the user of this program). This is reminiscent of the impredicative encodings of data types[2], with initial algebras having universal types and final coalgebras having existential types.

[1] *Finite semantics of polymorphism, complexity and the power of type fixpoints*, with Paolo Pistone, Thomas Seiller and Lorenzo Tortora de Falco. Available at `https://hal.archives-ouvertes.fr/hal-01979009/document`.

[2] To quote J.-Y. Girard (*The Blind Spot*, §12.B.2): "The second-order translations do invert polarities. [...] This inversion of polarities is due to deep reasons – so deep that they remain completely obscure."