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The 10th Asian Logic Conference was held September 1–6, 2008 in Kobe University, Japan. Major funding was provided by the Association for Symbolic Logic, the Japan Society for the Promotion of Science, the Inoue Foundation for Science, and Kobe University. The Program Committee consisted of Toshiyasu Arai(Chair), Jörg Brendle, Chi Tat Chong, Rod Downey, Qi Feng, Hirotaka Kikyo, and Hiroakira Ono. The members of the Local Organizing Committee were Mutsunori Banbara, Makoto Kikuchi(Chair), Hiroaki Minami, Ichiro Nagasaka, and Akira Suzuki.

The program included three four-hour tutorials, given by Peter Cholak, Greg Hjorth, and Byunghan Kim, and four one-hour plenary talks given by Jeremy Avigad, Justin Moore, Kazushige Terui and Yang Yue. There also were three parallel special sessions—one on model theory, non-classical logics, proof theory and constructive mathematics, one on recursion theory, and one on set theory—that together consisted of twenty-three invited thirty-minute talks. Twenty-three twenty-minute contributed talks were given.

There were 121 registered participants from Australia, Austria, China, Colombia, the Czech Republic, India, Israel, Japan, the Netherlands, New Zealand, Singapore, South Korea, Spain, the United Kingdom, and the United States.

Abstracts of the invited talks and contributed talks given (in person or by title) by members of the Association for Symbolic Logic follow.

For the Program Committee
TOSHIYASU ARAI

Abstracts of invited tutorial talks

► PETER CHOLAK, *The computably enumerable sets: A tutorial*.
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We will focus on the computably enumerable sets under inclusion, $\mathcal{E}$. By Scott Isomorphism Theorem, we know that for any countable structure there is an $\mathcal{L}_{\omega_1,\omega}$ formula, φ such that two elements are in the same orbit iff they both satisfy φ. We will explore local versions of this result within $\mathcal{E}$. The computably enumerable sets can also be explored in terms of Turing reducibility and jump classes or Turing complexity. Over the past years there have been many results relating Turing complexity, (elementary and non-elementary) definability and orbits within the structure $\mathcal{E}$. We will try to put these results into perspective. We will address what remains to be done and some possible approaches these problems.

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► GREG HJORTH, *Descriptive set theory, orbit equivalence, cost, and Borel equivalence relations*.
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The study of Borel and measurable equivalence relations has been considered from various points of view. I want to describe some of the interaction between researchers in Descriptive Set Theory and other fields such as Operator Algebras, Ergodic Theory, and Geometric Group Theory.

I will start by discussing Damien Gaboriau's revolutionary work on the concept of the "cost" of an equivalence relation. I will talk about some of the contributions to this area by Descriptive Set Theorists such as Alexander Kechris, Ben Miller, and myself. I will also try to describe, perhaps more informally, Sorin Popa's notions of "rigidity" in the context of equivalence relations, and how this, along with work in the theory of cost by Gaboriau and Russ Lyons, along with refinements of his ideas due to Adrian Ionna, finally led to Inessa Epstein's recent proof that every countable non-amenable group has continuum many orbit inequivalent measure preserving, ergodic, free, actions on standard Borel probability spaces.

► BYUNGHAN KIM, *Geometric simplicity theory*.
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In the series of tutorial lectures I will survey on recently developed Geometric Simplicity Theory. The class of simple theories, properly containing that of stable theories, is introduced by Shelah in [8], and a number of important algebraic simple structures are studied by Hrushovski and others. Then in B. Kim's thesis and consequent works with Pillay [4, 7, 3], it is revealed that the fundamental notion of forking is surprisingly well-behaved for (and within) simple theories, which greatly expands the scope of Stability Theory (the study of stable theories) to Simplicity Theory. After then the subject is rapidly grown and intensively studied by many leading researchers, and a book [9] by F. O. Wagner summarizing results up until year 2000 is published. In the 1st talk, I will start with these accounts.

Then during the last few years, the direction of research in Simplicity Theory moves for comprehending the combinatorial geometric side parallel to Geometric Stability Theory initiated and spurred by solving and understanding Zilber's principle for strongly minimal structures. The same questions and principle perfectly make sense in more general context of the simple theories, and for ω-categorical case, some of important results supplying new points of views are obtained [1] earlier, which I will speak in my 2nd talk.

For more general non ω-categorical case, the group configuration theorem should play a pivotal role to develop deeper Geometric Theory. Recently the theorem is achieved for simple theories [2, 5]. Namely it is proved that, under 4-amalgamation, the canonical hyperdefinable group acting on the hyperdefinable set can be obtained from a group configuration. As an application, when such a theory is non-trivial and modular($=$ 1-based), there is a hyperdefinable infinite vector space whose linear independence exactly comes from forking independence. This says Zilber's principle for 1-based case is more or less satisfied. This is the topic of the 3rd talk.

The generalized amalgamation is essential in showing the group configuration theorem. There has been a connection with the notion of n-simplicity, and recently very intriguing and unexpecting phenomena of the relationship are exposed [6]. The issue is also to do with the so-called the generalized elimination of imaginaries even for stable theories. I will speak about these in the last talk. Several research directions and open problems will be mentioned too. In particular, the more difficult part of Zilber's principle, namely constructing a field in a non-modular simple context is widely open. Even what field is supersimple is not known except a conjectural form that it should be pseudo algebraically closed.

[1] T. de Piro and B. Kim, *The geometry of 1-based minimal types*, ***Transactions of American Mathematical Society***, vol. 355 (2003), pp. 4241–4263.

[2] T. de Piro, B. Kim and J. Millar, *Constructiong the hyperdefinable group from the group configuration*, ***Journal of Mathematical Logic***, vol. 6 (2006), pp. 121–139.

[3] B. Hart, B. Kim and A. Pillay, *Coordinatisation and canonical bases in simple theories*, ***The Journal of Symbolic Logic***, vol. 65 (2000), pp. 293–309.

[4] B. Kim, *Forking in simple unstable theories*, ***Journal of the London Mathematical Society***, vol. 57 (1998), no. 2, pp. 257–267.

[5] ———, *Recovering the hyperdefinable group action in the group configuration theorem*, ***The Journal of Symbolic Logic*** (to appear).

[6] B. Kim, A. Kolesnikov and A. Tsuboi, *Generalized amalgamation and n-simplicity*, ***Annals of Pure and Applied Logic***, vol. 155, (2008), pp. 97–114.

[7] B. Kim and A. Pillay, *Simple theories*, ***Annals of Pure and Applied Logic***, vol. 88 (1997), pp. 149–164.

[8] S. Shelah, *Simple unstable theories*, ***Annals of Mathematical Logic***, vol. 19 (1980), pp. 177–203.

[9] F. O. Wagner, ***Simple theories***, Kluwer Academic Publishers, Dordrecht, 2000.

Abstracts of invited plenary talks

► JEREMY AVIGAD, *A formal system for Euclidean diagrammatic reasoning*.

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For more than two thousand years, Euclid's *Elements* was viewed as the paradigm of rigorous argumentation. But this changed in the nineteenth century, with concerns over the use of diagrammatic inferences and their ability to secure general validity. Axiomatizations by Pasch, Hilbert, and later Tarski are now taken to rectify these shortcomings, but proofs in these axiomatic systems look very different from Euclid's.

In this talk, I will argue that proofs in the *Elements*, taken at face value, can be understood in formal terms. I will describe a formal system with both diagram—and text-based inferences that provides a much more faithful representation of Euclidean reasoning. For the class of theorems that can be expressed in the language, the system is sound and complete with respect to Euclidean fields, that is, the semantics corresponding to ruler and compass constructions.

The system's one-step inferences are smoothly verified by current automated reasoning technology. This makes it possible to formally verify Euclidean diagrammatic proofs, and provides useful insight into the nature of mathematical proof more generally.

(This talk presents work carried out jointly with Ed Dean and John Mumma.)

► JUSTIN MOORE, *Structure within the class of Aronszajn lines*.

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An Aronszajn line is an uncountable linear order which does not contain an uncountable separable or scattered suborder. I will give an exposition of a number of results—some new and some old—concerning the relation of embeddability within the class of Aronszajn lines.

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► KAZUSHIGE TERUI, *Algebraic proof theory for nonclassical logics*.
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We outline an algebraic, systematic approach to the proof theory of nonclassical logics.

In the first part, we introduce a hierarchy on propositional formulas, which we call the substructural hierarchy in analogy with the arithmetical hierarchy. A wide range of logical axioms in the literature of fuzzy, intermediate and substructural logics can be naturally classified based on it. If an axiom is located at a low level of the hierarchy, then it can be (hyper)structuralized, i.e., presented as a structural rule of (hyper)sequent calculus. It is then amenable to proof theoretic analysis.

In the second part, we illustrate algebraization of some proof theoretic techniques. Typically, we reformulate proofs of cut-elimination and interpolation as sort of algebraic completion (like Dedekind's completion of the rational numbers into the real numbers). This gives rise to an algebraic criterion for cut-elimination, and also to a unified view on the logical interpolation property and the algebraic amalgamation property.

► YUE YANG, *Elementary Differences among Degree Structures*.
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In Recursion Theory, various degree structures $\mathcal{D} = (D, \leq)$ are studied. Often one can start with a reducibility, say $\leq_r$, which is a partial order on subsets of natural numbers; and study D—the collection of degrees which are equivalence classes modulo the induced equivalence relation $\equiv_r$, together with $\leq$ which is the induced partial order on degrees. If we naturally vary the domain D, we may obtain a nested collection of degree structures. One can ask whether there are elementary differences among the structures and whether one structure can be a Σ_n-elementary substructure (for some n) of the other. In this talk I will focus on a specific collection of structures $(D_n, \leq_T)$, where $D_n, n = 1, 2, \ldots$ are the so-called n-r.e.-degrees, which form the finite levels of Ershov hierarchies; and $\leq_T$ is the Turing reducibility. I will give a survey of earlier results and discuss some recent joint works with Slaman and Yu in this field.

Abstracts of invited talks in the Special Session on Model Theory, Non-classical logics, Proof Theory and Constructive Mathematics

► MICHAEL BEESON, *Constructive geometry*.
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We consider that Euclidean geometry consists of straightedge-and-compass constructions and rigorous reasoning about the results of those constructions. This viewpoint leads to a consideration of algebras of constructions, and interesting questions as to the definability of some constructions in terms of others. A consideration of the relation of the Euclidean "constructions" to "constructive mathematics" leads to the development of a first-order theory EGC of the "Elementary Geometric Constructions", which can serve as an axiomatization of Euclid rather close in spirit to the *Elements* of Euclid. We apply the methods of modern metamathematics to this theory, showing that if EGC proves an existential theorem, then the object proved to exist can be constructed from parameters, using the basic constructions of EGC. Several variants of EGC are considered and their relationships discussed. Close attention is paid to the question of continuous dependence on parameters of constructed objects.

► DOUGLAS S. BRIDGES, *Dynamical systems, compact orbits, and continuous group homomorphisms*.
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A fundamental classical theorem states that every compact orbit of a dynamical system is periodic. We first discuss the constructively inadmissible aspects of the standard classical proof, to reveal some of the non-trivial problems associated with the search for a (Bishop-style) constructive proof. We then strip away the apparatus of dynamical systems, to reveal the problem as one about continuous epimorphisms from **R** to metric abelian groups. There are then two—classically equivalent but constructively independent—theorems:

THEOREM 1. *Let θ be a continuous homomorphism of* **R** *onto a compact abelian group G. Then there exists $\tau \neq 0$ such that $\theta(\tau) = 0$.*

THEOREM 2. *Let θ be a continuous one-one homomorphism of* **R** *onto a complete metric abelian group G, such that the set $S_1 \equiv \{\theta(t) : |t| > 1\}$ is located in G. Then G is* **noncompact**, *in the sense that for each compact $K \subset G$, the metric complement*

$$G - K \equiv \{x \in G : \rho(x, K) > 0\}$$

is inhabited.

The proof in each case uses Baire's category theorem. For Theorem 2 we need to develop a number of classically vacuous, constructively interesting (and even amusing) preliminary results about injective mappings from **R** onto a complete metric space.

This is joint work with Matt Hendtlass (and Fred Richman).

► KOICHIRO IKEDA, *On saturated generic structures*.
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Let L be a finite relational language and **K** a class of finite L-structures with non-negative predimension. Then a countable L-structure M is said to be **K**-generic, if (i) every finite $A \subset M$ belongs to **K**; (ii) for every finite $A \leq B \in \mathbf{K}$ with $A \leq M$ there is a copy B' of B over A with $B' \leq M$; (iii) for every finite $A \subset M$, $\mathrm{cl}_M(A)$ is finite. Hrushovski's pseudoplanes and strongly minimal sets are generic and saturated. In my talk I introduce some results on saturated generic structures.

► MAKOTO KANAZAWA, *A lambda calculus characterization of MSO definable tree transductions*.
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Bloem and Engelfriet [1] have shown that the tree transductions definable in monadic second-order logic are exactly those computed by attributed tree transducers with look-ahead. We obtain a third characterization of this class of tree transductions in terms of "homomorphisms" that map symbols in a ranked alphabet to lambda terms. Such homomorphisms are powerful enough to be able to express the composition closure of macro tree

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transductions. The class of MSO definable tree transductions is captured by those homomorphisms whose image is restricted to lambda terms that are *almost linear* in the sense of Kanazawa [2] (combined with a suitable notion of look-ahead).

[1] RODERICK BLOEM AND JOOST ENGELFRIET, *A comparison of tree transductions defined by monadic second order logic and by attribute grammars*, ***Journal of Computer and System Sciences***, vol. 61 (2000), pp. 1–50.

[2] MAKOTO KANAZAWA, *Parsing and generation as Datalog queries*, ***Proceedings of the 45th annual meeting of the Association for Computational Linguistics*** (Prague, Czech Republic), Association for Computational Linguistics, 2007, pp. 176–183.

► THOMAS SCANLON, *(Non)-bi-interpretability of finitely generated rings with* $\mathbb{N}$.
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To demonstrate a conjecture of Pop [2], we showed that an infinite finitely generated field must be parametrically bi-interpretable with $\mathbb{N}$ [3]. Khelif used this result to show that every finitely generated commutative ring is quasi-finitely axiomatizable within the class of Noetherian rings [1], but left open the question of bi-interpretability with $\mathbb{N}$. Taking as a starting point the observation that $\mathbb{Z} \times \mathbb{Z}$ is *not* bi-interpretable with $\mathbb{N}$ and the less obvious observation that there do exist nonstandard models of true arithmetic possessing nontrivial derivations, we characterize those finitely generated commutative rings which are bi-interpretable with $\mathbb{N}$. (This is a report on joint work with Matthias Aschenbrenner.)

[1] A. KHELIF, *Bi-interprétabilité et structures QFA: étude de groupes résolubles et des anneaux commutatifs*, ***Comptes Rendus Mathématiques. Académie des Sciences. Paris***, vol. 345 (2007), no. 2, pp. 59–61.

[2] F. POP, *Elementary equivalence versus isomorphism*, **Inventiones Mathematicae**, vol. 150 (2002), no. 2, pp. 385–408.

[3] T. SCANLON, *Infinite finitely generated fields are biinterpretable with* $\mathbb{N}$, ***Journal of the American Mathematical Society***, vol. 21 (2008), no. 3, pp. 893–908.

► FREEK WIEDIJK, *Avoiding state with infinite contexts*.
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Proof assistants are computer programs that verify the correctness of mathematics. Such a program generally has a *state*, of which an essential part is a table that holds the *defined symbols* that so far have been introduced in the mathematics. Logically that part of the state corresponds to the *contexts* that occur in the logical derivations that represents the mathematics that is being verified.

In most logical systems free variables in the statements are not accounted for in the contexts of the judgements. However, in type theory it is customary to administer these variables in the contexts as well. In this talk we will go to the other extreme and present a version of type theory in which there are *no* explicit contexts. All variables (even the variables that correspond to assumptions) are treated as free. In some sense they are still being bound, but in an *infinite* context.

This logical system inspires an implementation approach for LCF style proof assistants where the logical core of the system no longer needs state. (For this approach it is essential that the programming language supports testing for *pointer equality*, to allow fast equality checking.)

We present two prototypes of a logical core for a proof assistant using this approach. The first is a version of John Harrison's HOL Light system in which state has been removed from the logical core. This change allows the system to support 'undoing' definitions in a logically sound way. The second is an implementation of a Pure Type System in LCF style.

► ALAN R. WOODS, *Number theoretic consequences of counting in bounded arithmetic*.
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$I\Delta_0$ is the axiom system similar to Peano Arithmetic, but with induction hypotheses restricted to being Δ_0 formulas, i.e., arithmetic formulas having only *bounded quantifiers* of the form $\exists y \le x$, $\forall y \le x$. There is a long list of theorems of elementary number theory for which no proof in $I\Delta_0$ is known, and yet which are provable if, for a suitably chosen Δ_0 formula $\varphi(x, \vec{y})$, one adds a "new" function symbol $c_\varphi(x, \vec{y})$ together with an axiom asserting

$$c_\varphi(0, \vec{y}) = 0,$$
$$c_\varphi(x+1, \vec{y}) = \begin{cases} c_\varphi(x, \vec{y}) + 1 & \text{if } \varphi(x+1, \vec{y}), \\ c_\varphi(x, \vec{y}) & \text{otherwise,} \end{cases}$$

and allows induction on bounded quantifier formulas containing c_φ. This *census function* c_φ ensures that for each choice of the parameters $\vec{y}$, the number $c_\varphi(x, \vec{y})$ provides a well defined notion of *cardinality* for the Δ_0 definable set $\{j : 1 \le j \le x \wedge \varphi(j, \vec{y})\}$.

As an example, take $\xi(x, y, e)$ to be the number of primes p less than or equal to x for which the integer part $[y/p^e]$ is odd. Let $I\Delta_0(\xi)$ be $I\Delta_0$ augmented by ξ and its recursive definition. Then:

THEOREM 1 (Woods and Cornaros [2])). $I\Delta_0(\xi) \vdash$ *Bertrand's Postulate*.

(Recall that *Bertrand's Postulate* asserts that for every $n \ge 1$ there is some prime number p satisfying $n < p \le 2n$. However no $I\Delta_0$ proof of the existence of arbitrarily large primes is known.)

Other examples of this phenomenon will be surveyed. Typically the theorems in question are provable in $I\Delta_0$ augmented by a "combinatorial principle" such as the Δ_0 *Pigeonhole Principle* or the Δ_0 *Equipartition Principle*. For both of these principles it is known that every instance can be proved by introducing the census function c_φ of an appropriately chosen Δ_0 formula φ. Particular mention will be made of Jeřábek's recent proof [1] of the *Quadratic Reciprocity Law* using the Δ_0 *Equipartition Principle*, and of some other consequences of his advance.

[1] EMIL JEŘÁBEK, *Abelian groups and quadratic residues in weak arithmetic*, Mathematics Institute of the Czech Academy of Sciences, preprint, June 2008.

[2] ALAN R. WOODS AND CH. CORNAROS, *On bounded arithmetic augmented by the ability to count certain sets of primes*, ***The Journal of Symbolic Logic***, to appear.

Abstracts of invited talks in the Special Session on Set Theory

► DAVID ASPERÓ, *Lightface definable well-orders of $H(\omega_2)$ and forcing axioms*.
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I will survey some recent results on the problem of forcing models in which there is a lightface definable well-order of $H(\omega_2)$ (that is, definable over the structure $\langle H(\omega_2), \in\rangle$ by a parameter-free formula) and a strong forcing axiom holds.

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► LONGYUN DING, *Metrics on free groups and Polishable subgroups*.
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In this talk we will give a survey of recent results on two topics.

The first topic is to study metrics on $F(X)$, the free group over a metric space X. Firstly, we will review the Graev metric on $F(X)$. This metric is two-sided invariant. Furthermore, while X is the Baire space $\mathcal{N}$, the completion of $F(\mathcal{N})$ under the Graev metric turns out to be a surjectively universal for all two-sided invariant metric Polish groups. Secondly, to study the existence of surjectively universal Polish groups, we generalize Graev metrics to obtain more metrics on the free group $F(\mathcal{N})$. We will show that if there exist some surjectively universal Polish groups, then we can find such a universal group from completions of $F(\mathcal{N})$ under these new metrics.

The second topic is the existence of Polishable subgroups of arbitrarily high Borel rank in every uncountable Polish group. We will review the history of the study on this topic. For uncountable abelian Polish groups or uncountable solvable Polish groups, the question was answered in the affirmative. In the end, we will present several equivalent conditions of the question for the general case.

[1] L. Ding and S. Gao, *New metrics on free groups*, ***Topology and its Applications***, vol. 154 (2007), no. 2, pp. 410–420.

[2] ———, *Grave metrics groups and Polishable subgroups*, ***Advances in Mathematics***, vol. 213 (2007), no. 2, pp. 887–901.

[3] I. Farah and S. Solecki, *Borel subgroups of Polish groups*, ***Advances in Mathematics***, vol. 199 (2006), no. 2, pp. 499–541.

[4] M. I. Graev, ***Free topological groups***, American Mathematical Society Translations, vol. 35 (1951), 61 pp.

[5] G. Hjorth, *Subgroups of abelian Polish groups*, ***Set theory, Centre de Recerca Matemàtica Barcelona, 2003–2004***, Trends in Mathematics, (J. Bagaria and S. Todorcevic, editors), Birkhäuser, Basel, 2006, pp. 297–308.

► STEFAN GESCHKE, *Automorphisms of* $\mathcal{P}(\omega)/\texttt{fin}$ *and related structures*.
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An automorphism of $\mathcal{P}(\omega)/\texttt{fin}$ is *trivial* if it is induced by a bijection between two cofinite subsets of ω. It is well known that the existence of nontrivial automorphisms is independent of ZFC. The typical example of a trivial automorphism is the socalled shift s that is induced by the map that maps every natural number to its immediate successor.

It is an open question whether the structures $(\mathcal{P}(\omega)/\texttt{fin}, s)$ and $(\mathcal{P}(\omega)/\texttt{fin}, s^{-1})$ can be isomorphic. They are not isomorphic via a trivial automorphism of $\mathcal{P}(\omega)/\texttt{fin}$. In particular, the two structures are not isomorphic if all automorphisms of $\mathcal{P}(\omega)/\texttt{fin}$ are trivial.

We mention some partial results concerning this question and review what is known for other, related structures such as the group $S(\omega)/FS$, the symmetric group on ω factored by the normal subgroup of permutations with finite support, and the Calkin algebra $C(H)$, the algebra $B(H)$ of bounded operators on a separable Hilbert space H factored by the ideal $K(H)$ of compact operators.

► MENACHEM KOJMAN, *Extensions of the infinite Ramsey theorem*.
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In this talk I will survey several old and new extensions of the infinite Ramsey thoerem. I will describe induced theorems for perfect graphs and symmetric induced theorems, in which the monocolored induced target object satisfies that each of its automorphisms extends to an automorphism of the colored source object

► MASAHIRO SHIOYA, *A model of a saturated ideal*.
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In the pioneering work Kunen constructed a model in which there is an ω_2-saturated ideal on ω_1. In this talk we present a new such model. We expect that the method can be applied to other problems, in particular Chang's conjecture.

► SLAWOMIR SOLECKI, *Isometry groups of metric spaces*.
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The talk will be on representations of Polish groups as isometry groups of separable metric spaces. I will present a theorem, joint with Malicki, characterizing locally compact, second countable groups as the full isometry groups of proper separable metric spaces. I will describe the context for this result, in particular, theorems of Gao and Kechris and of Melleray giving characterizations of Polish groups and of compact, second countable groups as full isometry groups of appropriate classes of separable metric spaces. Our theorem completes this series of earlier results.

► TERUYUKI YORIOKA, *Two properties which come from Aronszajn trees*.
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Todorčević introduced some fragments of $\mathsf{MA}_{\aleph_1}$ and gave many applications, but many problems on implications between these fragments have been left open. We will focus on the following three fragments here: $\mathcal{K}_2$ means that every ccc forcing notion has the property K. $\mathcal{K}_2'$ means that every ccc partition $K_0 \cup K_1 = [\omega_1]^2$ has an uncountable K_0-homogeneous set. $\mathcal{C}^2$ means that any product of ccc forcing notions is also ccc. We notice that $\mathsf{MA}_{\aleph_1}$ implies all three, and $\mathcal{K}_2$ implies $\mathcal{K}_2'$ and $\mathcal{C}^2$, however it is unknown whether any other implications hold in ZFC. For example, the following problems have not been settled yet: Problem 1: Does $\mathcal{K}_2'$ imply $\mathcal{K}_2$? Problem 2: Does $\mathcal{C}^2$ imply $\mathsf{MA}_{\aleph_1}$? In this talk, we consider these two problems by restricting to classes of partitions and forcing notions. To do this, we introduce two properties which come from Aronszajn trees: The anti-rectangle refining property (arec) and the property anti-$\mathsf{R}_{1,\aleph_1}$.

A forcing notion $\mathbb{P}$ has the arec if $\mathbb{P}$ is uncountable and

$$\forall I, J \in [\mathbb{P}]^{\aleph_1}\ \exists I' \in [I]^{\aleph_1}\ \exists J' \in [J]^{\aleph_1}\ \forall p \in I'\ \forall q \in J'\ (p \perp_{\mathbb{P}} q).$$

A forcing notion $\mathbb{P}$ has the property anti-$\mathsf{R}_{1,\aleph_1}$ if $\mathbb{P}$ is uncountable and for any large enough regular κ,

$$\forall \text{ countable } M \prec H(\kappa) \text{ with } \mathbb{P} \in M\ \forall I \in [\mathbb{P}]^{\aleph_1} \cap M\ \forall p \in \mathbb{P} \setminus M$$
$$\exists I' \in [I]^{\aleph_1} \cap M\ \forall q \in I'\ (p \perp_{\mathbb{P}} q).$$

We note that for any ω_1-tree T, T is Aronszajn iff T has the arec as a forcing notion iff T has the anti-$\mathsf{R}_{1,\aleph_1}$ as a forcing notion. Using the anti-rectangle refining property, we give an affirmative answer of Problem 1 for partitions with the rectangle refining property, which has been introduced by Larson–Todorčević to solve Katětov's problem. And using the property anti-$\mathsf{R}_{1,\aleph_1}$, we give a negative answer of Problem 2 for the class of forcing notions related to the anti-$\mathsf{R}_{1,\aleph_1}$.

We can also show that if $\mathbb{P}$ has the arec or the anti-$\mathsf{R}_{1,\aleph_1}$, then $a(\mathbb{P})$, which consists of finite antichains on $\mathbb{P}$, ordered by the reverse inclusion, does not add random reals. This forcing is quite different from known ccc forcing notions not adding random reals.

► YASUO YOSHINOBU, *Forcing axioms and closure properties of posets*.
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In the theory of forcing, versions of closure properties of posets, like 'closed', 'directed closed', 'strategically closed', *etc.*, have been investigated. Some of these properties can be distinguished from each other, by observing how large fragments of Martin's Maximum (MM) are preserved under forcing with posets with them. For example, MM is preserved under forcing with any ω_2-directed closed posets, whereas there exists an ω_2-closed poset which destroys MM, as was observed by König and the author ([2]).

In this session we will first review some previous results of this type, which are due to König and the author ([1] and [2]), and then mention some recent developments in this area.

[1] B. König and Y. Yoshinobu, *Fragments of Martin's Maximum in generic extensions*, ***Mathematical Logic Quarterly***, vol. 50 (2004), no. 3, pp. 296–302.

[2] ———, *Kurepa trees and Namba forcing*, submitted.

Abstracts of invited talks in the Special Session on Recursion Theory

► JOHANNA N. Y. FRANKLIN, *Relativizations of randomness and genericity notions*.
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The set of reals that are low for a particular randomness notion is a subset of the set of reals that are bases for that randomness notion. For Martin–Löf randomness, these two sets coincide. Here, we show that this is not the case for Schnorr randomness and weak 1-genericity and that furthermore, the bases for each are precisely the reals A such that $A \geq_T K$.

We further show that this set of reals may also be characterized as the reals A such that every real that is Schnorr random relative to A is Martin–Löf random. This leads us to define "highness" for pairs of randomness notions as a dual to the more standard concept of lowness for such pairs.

This is joint work with Frank Stephan and Yu Liang.

► NOAM GREENBERG AND JOSEPH S. MILLER, *A minimal degree of positive effective Hausdorff dimension*.
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Using slow-growing diagonally non-recursive functions, we show that there is a set $X \in 2^\omega$ of minimal Turing degree and positive effective Hausdorff dimension.

► BJOERN KJOS-HANSSEN, *Asarin's theorem on incompressible random walk*.
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Asarin, in a paper with Pokrovskii (1986) showed that for $\gamma = \frac{1}{10}$, a path of Brownian motion $x(t)$ can be approximated to within $n^{-\gamma}$ in uniform distance by scaled and interpolated

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random walks of length n that are incompressible in the sense of Kolmogorov complexity. We show that this can be improved to $\gamma = 1/6 - \varepsilon$ but not to $\gamma = 1/2$. We are working on extending the result to $\gamma = 1/4 - \varepsilon$ and $\gamma = 1/2 - \epsilon$.

▶ MASAHIRO KUMABE, *Fixed point free minimal degrees*.
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Let φ_e be the e-th Turing computable function. A function f is called fixed point free if for each e, $\varphi_e \neq \varphi_{f(e)}$. We show that there is a fixed point free minimal Turing degree.

▶ KAZUYUKI TANAKA, *Determinacy, Ramsey property and* Π^1_2*-comprehension*.
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Recently, remarkable progress has been made in reverse mathematics concerning Π^1_2-comprehension. For instance, Mummert and Simpson found a metrization theorem of topology to be equivalent to Π^1_2-comprehension. In this talk, we will elucidate some elaborate relations among determinacy, Ramsey property, inductive definitions and thier variants in respect of Π^1_2-comprehension.

▶ WEI WANG, *Constructing hyperimmune-free sets for admissible ordinals*.
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In classical recursion theory, we call a set A of natural numbers *hyperimmune* if no strong array always intersects A. On the other hand, a set X is *hyperimmune-free* (HIF for short) if it is Turing equivalent to no hyperimmune set, equivalently every function recursive in X is dominated by some recursive function. The typical construction of a HIF set is by closed set forcing, and closely resembles the construction of a minimal degree.

Recently, Chitat Chong and I [1] generalized HIF to admissible ordinals. To this end, we introduced trees whose nodes are definable in a uniformly way. Using these trees, we constructed non-trivial HIF sets for countable admissible ordinals. With priority ordering coded into roots of trees, we also constructed non-trivial HIF sets for Σ_2-admissible ordinals. Furthermore we showed that the cardinality of HIF sets for cardinals depends on whether there exists effective Kurepa families. The technique behind this cardinality question can also be applied to prove non-existence result of non-trivial HIF sets for certain cardinals.

In this talk, I will sketch the technical aspects of constructing non-trivial HIF sets and also of calculating cardinality of HIF sets.

[1] Chitat Chong and Wei Wang, *Hyperimmune-free degrees beyond* ω, to appear.

▶ GUOHUA WU, *The nonisolating degrees are nowhere dense in the computably enumerable degrees*.
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A d.c.e. degree $\mathbf{d}$ is *isolated by a c.e. degree* $\mathbf{a}$, if $\mathbf{a} < \mathbf{d}$ is the greatest c.e. degree below $\mathbf{d}$. $(\mathbf{a}, \mathbf{d})$ is called an isolation pair. Say that $\mathbf{d}$ is *isolated* if there is a c.e. degree $\mathbf{a}$ isolating $\mathbf{d}$. Both the isolated degrees, and hence, the isolating degrees, and the nonisolated d.c.e. degrees are dense in the c.e. degrees.

We consider the distribution of the nonisolating degrees. Salts proved in his thesis that there is an interval of computably enumerable degrees, each of which isolates a d.c.e. degree. As a consequence, unlike the isolating degrees, the nonisolating degrees are not dense in the

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c.e. degrees. In this talk, we will show that such isolating intervals are dense in the c.e. degrees. Therefore, the nonisolating degrees are nowhere dense in the computably enumerable degrees.
This is a joint paper with Cenzer and LaForte.

► LIANG YU, *Martin's Conjecture for Π^1_1 functions*.
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We investigate Martin's conjecture for Π^1_1 functions. This is joint work with Chong and Wang.

Abstracts of contributed talks

► YOSHIHIRO ABE, *Kunen's theorem on normal measures without the partition property*.
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It was shown by Kunen that the least cardinal λ such that $\mathcal{P}_\kappa\lambda$ carries a normal measure without the partition property is Π^2_1-indescribable.
We just extend this to prove:

The least cardinal λ such that cf$\lambda \geq \kappa$ and $\mathcal{P}_\kappa\lambda$ carries a normal $(\lambda, 2)$-distributive ideal without the partition property is Π^2_1-indescribable.

[1] DiPrisco–Zwicker, *Flipping properties and supercompact cardinals*, ***Fundamenta Mathematicae***, vol. 109 (1980), no. 1, pp. 31–36.

[2] Kunen–Pelletier, *On a combinatorial property of Menas related to the partition property for measures on supercompact cardinals*, ***The Journal of Symbolic Logic***, vol. 48 (1983), no. 2, pp. 475–481.

► JOSEF BERGER, *The fan theorem for c-bars*.
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Let $\{0,1\}^*$ denote the set of all finite binary sequences. A subset B of $\{0,1\}^*$ is a *c-set* if there is a decidable subset D of $\{0,1\}^*$ such that for all $u \in \{0,1\}^*$ we have

$$u \in \mathrm{B} \leftrightarrow \forall w \in \{0,1\}^* \, (u * w \in \mathrm{D}),$$

where $u * w$ denotes the concatenation of u and w. This gives rise to the following version of the fan theorem.

c-FAN Every bar which is a c-set is a uniform bar.

The letter 'c' in these expressions indicates that this notion of complexity is related to continuity; we have shown in [1] that, in Bishop-style constructive mathematics [2], c-FAN is equivalent to the statement 'every pointwise continuous function $F\colon \{0,1\}^{\mathbb{N}} \to \mathbb{N}$ is uniformly continuous'.

In this talk we will prove that in Simpson-style reverse mathematics [3], c-FAN is equivalent to the arithmetical comprehension axiom. We will further discuss some consequences of this result.

[1] Josef Berger, ***The logical strength of the uniform continuity theorem***, Lecture Notes in Computer Science, vol. 3988, Springer, 2006, pp. 35–39.

[2] Errett Bishop and Douglas Bridges, ***Constructive analysis***, Grundlehren der mathematischen Wissenschaften, Springer-Verlag, 1985.

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[3] Stephen G. Simpson, ***Subsystem of second order arithmetic***, Perspectives in Mathematical Logic, Springer-Verlag, 1999.

► ANDREW BROOKE-TAYLOR AND DAMIANO TESTA, *A new zero-one law for simplicial complexes*.

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In the 1970's Blass and Harary [1] proved a zero-one law for simplicial complexes, analogous to that for graphs. We show, however, that another zero-one law is possible, using a different but nonetheless very natural measure on the set of simplicial complexes with a given number of vertices. This is achieved by considering the Fraïssé limit of the class of finite simplicial complexes, a structure which is analogous to the random graph in various respects.

[1] Andreas Blass and Frank Harary, *Properties of almost all graphs and complexes*, ***Journal of Graph Theory***, vol. X (1979), pp. 225–240.

► NAOHI EGUCHI, *A term-rewriting characterization of PSPACE*.

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We give a term-rewriting characterization of the class **FPS** of the functions computable in polynomial space. The main novelty is the Bellantoni–Cook style safe representation. Restricting the operation of primitive recursion, Bellantoni and Cook [2] introduce the scheme of safe recursion based on a principle of normal/safe variable-separation. It is shown that the polynomial-time computable functions are captured by safe recursion on notation.

In this talk, we show that **FPS** is characterized by a natural safe variation of nested recursion. Safe nested recursion is introduced by Arai and the author. In [1] it is shown that the exponential-time computable functions are captured by safe nested recursion on notation (SNRN). In [3] the SNRN scheme is treated in a term-rewriting framework. Another safe representation of **FPS** is known. The corresponding term-rewriting characterization is given by Oitavem [4].

We introduce a class N as the smallest class containing specific initial functions, and closed under safe composition and SNRN. Let N_{normal} be the subset of N containing only the functions over normal variables. Then the main theorem runs as follows.

Theorem 1. $N_{normal} =$ **FPS**.

To prove $N_{normal} \subseteq$ **FPS**, we consider a rewrite system R_N induced by N. We show that the term-rewriting relation $\rightarrow_{R_N}$ defined from R_N yields algorithms running in polynomial space.

[1] Toshiyasu Arai and Naohi Eguchi, *A new function algebra of EXPTIME functions*, to appear.

[2] Stephen Bellantoni and Stephen Cook, *A new recursion-theoretic characterization of the polytime functions*, ***Computational Complexity***, vol. 2 (1992), no. 2, pp. 97–110.

[3] Naohi Eguchi, *A lexicographic path order with slow growing derivation bounds*, submitted.

[4] Isabel Oitavem, *A term rewriting characterization of the functions computable in polynomial space*, ***Archive for Mathematical Logic***, vol. 42 (2002), no. 1, pp. 35–47.

► DAISUKE IKEGAMI, DAVID DE KLOET AND BENEDIKT LÖWE, *Consequences of Real Blackwell Determinacy*.
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Blackwell games are a variant of infinite games with imperfect information invented by Blackwell [1] in 1969. While Gale–Stewart games are infinite games with perfect information and have been deeply investigated, Blackwell games have not been researched so much as Gale–Stewart games. In 1998, Martin [3] proved that Axiom of Determinacy (AD) implies Axiom of Blackwell Determinacy (Bl–AD) and conjectured that the converse is true. Although this is still open, in 2003, Martin, Neeman and Vervoort [4] proved that they are equivalent if we assume $V = L(\mathbb{R})$, which immediately establishes the equiconsistency between AD and Bl–AD. In this talk, we investigate Axiom of Real Blackwell Determinacy (Bl–$\text{AD}_{\mathbb{R}}$), the Blackwell version of Axiom of Real Determinacy ($\text{AD}_{\mathbb{R}}$) introduced by Löwe [2], and show that Bl–$\text{AD}_{\mathbb{R}}$ implies that $\mathbb{R}^{\#}$ exists. Combining the above result, this result shows that Bl–$\text{AD}_{\mathbb{R}}$ implies the consistency of AD, especially that the consistency strength of Bl–$\text{AD}_{\mathbb{R}}$ is strictly stronger than that of AD.

[1] DAVID BLACKWELL, *Infinite G_δ games with imperfect information*, ***Matematwyki Applicationes Mathematicae***, vol. 10 (1969), pp. 99–101.

[2] BENEDIKT LÖWE, *Extensions of the axiom of Blackwell determinacy*, Technical report, Institute for Logic, Language and Computation, Universiteit van Amsterdam, 2005.

[3] DONALD A. MARTIN, *The determinacy of Blackwell games*, ***The Journal of Symbolic Logic***, vol. 63 (1998), no. 4, pp. 1565–1581.

[4] DONALD A. MARTIN, ITAY NEEMAN, AND MARCO VERVOORT, *The strength of Blackwell determinacy*, ***The Journal of Symbolic Logic***, vol. 68 (2003), no. 2, pp. 615–636.

[5] ROBERT M. SOLOVAY, *The independence of DC from AD*, ***Cabal Seminar 76–77*** (***Proceedings of the Caltech–UCLA Logic Seminar, 1976–77***), vol. 689, Lecture Notes in Mathematics, pp. 171–183, Springer, Berlin, 1978.

► MASAHIRO KUMABE AND TOSHIO SUZUKI, *Weak randomness, genericity and Boolean decision trees*.
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We regard a sequence of copies of a randomized Boolean decision tree (AND–OR tree) as a mapping of a given oracle X (of leaves) to another oracle Y (of roots), and we ask which weak-randomness property (of the leaf oracle) is conserved by such a mapping. The following property P_i $(i = 1, 2)$ of X is not necessarily shared by Y. P_1 "being Martin–Löf 1-random", P_2 "being Martin–Löf ε-random for some ε such that $0 < \varepsilon < 1$". We show that the following property P_3 of X is conserved in such a mapping of X to Y. P_3 "being r-generic in the sense of Dowd [1] for every positive integer r" (that is, for each r, the forcing complexity of the r-query tautologies with resepct to X is bounded by a polynomial; this property is defferent from r-genericity of arithmetical forcing). It is known that the class $\mathcal{D}$ of all oracles having property P_3 has Lebesgue measure one [1]. We show that $\mathcal{D}$ is closed under p-time bounded-truth-table reductions of the following properties: (1) Different inputs correspond to disjoint sets of queries, and (2) Every truth table is an onto Boolean function $\{0, 1\}^n \to \{0, 1\}$, where n is the norm.

[1] M. Dowd, *Generic oracles, uniform machines, and codes*, ***Information and computation***, vol. 96 (1992), pp. 65–76.

► GYESIK LEE, *Well-partial-orderings and hierarchies of binary trees*.
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The notion of well-partial-ordering appeared around 1950. After the celebrated work by Higman [3], this simple notion has found a large number of applications in the fields of algebras, combinatorics, mathematical logic, and computer science. One of the most elegant is Kruskal's theorem.

An important thing about well-partial-orderings is the existence of maximal order types, i.e., the greatest order type of linearizations, cf. [1]. This has an immediate consequence that well-partial-orderings are directly connected with ordinal notation systems. This is because of the equivalence of well-partial-orderedness and well-orderedness of ordinals. Since Gentzen's consistency proof of Peano arithmetic, it is well known that well-orderedness of large ordinals is independent of some logical systems. For example, that of the ordinal ε_0 is independent of the systems PA and ACA_0. This implies that the well-partial-orderedness of any well-partial-ordering of maximal order type ε_0 is independent too. An example is $(\mathcal{B}, \trianglelefteq)$, the set of binary trees with the homeomorphic embedding. Higman [3] showed that $(\mathcal{B}, \trianglelefteq)$ is a well-partial ordering, and in an unpublished paper, de Jongh showed that its maximal order type is ε_0.

Here we introduce a new proof of the fact mentioned just before by considering the set of binary trees as a cumulative hierarchy of well-partial-orderings. We use Higman's lemma to decide their maximal order types. Furthermore, Dershowitz's recursive path ordering [2] will be used for the decision of the order types of the sets from the hierarchy when they are canonically linearized. Here *canonically linearized* means that they are sub-orderings of a celebrated well-ordering in proof theory. Finally we characterize $\omega_n(k)$ using some sets of binary trees, where $\omega_0(k) = k$ and $\omega_{n+1}(k) = \omega^{\omega_n(k)}$.

[1] Dick H. J. de Jongh and Rohit Parikh, *Well-partial orderings and hierarchies*, **Nederlandse Akademie van Wetenschappen. Proc. Ser. A 80 = Indagationes Mathematicae**, vol. 39, no. 3, pp. 195–207, 1977.

[2] Nachum Dershowitz, *Orderings for term-rewriting systems*, ***Theoretical Computer Science***, vol. 17, no. 3, pp. 279–301, 1982.

[3] Graham Higman, *Ordering by divisibility in abstract algebras*, ***Proceedings of the London Mathematical Society***, 3rd series vol. 2, pp. 326–336, 1952.

► HEIKE MILDENBERGER AND LYUBOMYR ZDOMSKYY, *L-spaces under the P-ideal dichotomy*.
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We consider the effect of some consequences of the Proper Forcing Axiom on the properties of L-spaces. In particular, we extend two classical facts about L-spaces due to Szentmiklóssy and Kunen respectively which they proved under MA_{ω_1}. We assume $\mathfrak{p} > \omega_1$ and Abraham and Todorcevic's P-ideal dichotomy principle and derive strengthenings of the facts. Then we show that $\mathfrak{p} > \omega_1$ and the dichotomy principle for P-ideals that have at most $\aleph_1$ generators together with the two classical facts and not MA_{ω_1} is consistent relative to ZFC.

▶ TADATOSHI MIYAMOTO, *A coding and a strongly inaccessible cardinal*.
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We show an equiconsistency between a theory in which a principle decides the value of the continuum to be the second uncountable cardinal by coding every subset of the first uncountable cardinal and a theory in which a strongly inaccessible cardinal exists.

[1] J. Moore, *Set mapping reflection*, ***Journal of Mathematical Logic***, vol. 5 (2005), no. 1, pp. 87–97.

[2] D. Velleman, *Simplified morasses*, ***The Journal of Symbolic Logic***, vol. 49 (1984), no. 1, pp. 257–271.

▶ TAKAKO NEMOTO, *Weak weak König's lemma in constructive mathematics*.
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In classical reverse mathematics, it is know that weak König's lemma (WKL), which asserts every 0–1 labeled tree with branches of any finite length has an infinite path, implies many basic property of continuous function, e.g., uniform continuity of continuous function on [0, 1].

In [4], the countable additivity of Lebesgue measure is equivalent to the following:

Weak weak König's lemma (WWKL): If 0–1 labeled tree T has no infinite path, then

$$\lim_{n\to\infty} \frac{\sharp\{t \in T : \mathit{length}(t) = n\}}{2^n} = 0.$$

WKL has the following intuitionistic counterpart:

Brouwer's fan theorem (BFT): Every detachable 0–1 labeled tree without path is bounded.

[2] proved that, on constructive setting, BFT is equivalent to the assertion "Every positive uniformly continuous function on [0, 1] has a positive infimum."

We can check that WWKL is weaker than BFT even in constructive setting, as BFT asserts that, for every 0–1 labeled tree T without path, there exists n such that $\sharp\{t \in T : \mathit{length}(t) = n\} = 0$.

We prove the following weaker equivalence:

Theorem 2. *The following assertions are equivalent on constructive setting.*

- *Weak weak König's lemma*
- *Every positive, uniformly continuous function on* [0, 1] *satisfies*

$$\lim_{\delta\to 0} \mu(\{x \in [0,1] : f(x) < \delta\}) = 0.$$

[1] Erret Bishop and Henry Cheng, *Constructive measure theory*, ***Memoirs of the American Mathematical Society***, vol. 116, 1972.

[2] William Julian and Fred Richman, *A uniformly continuous function on* [0, 1] *that is everywhere different from its infimum*, ***Pacific Journal of Mathematics***, vol. 111 (1984), no. 2, pp. 333–340.

[3] Stephen G. Simpson, ***Subsystems of second order arithmetic***, Springer, 1991.

[4] Xiaokang Yu and Stephen G. Simpson, *Measure theory and weak König's lemma*, ***Archive for Mathematical Logic***, vol. 30 (1990), pp. 171–180.

► MASANAO OZAWA, *Quantum set theory: Transfer principle and modal interpretation*.
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Quantum set theory (QST) was introduced by Takeuti [3] in 1981 by constructing a model of set theory based on the quantum logic, the lattice of closed subspaces in a Hilbert space, introduced by Birkhoff and von Neumann. A primary problem is to figure out what modifications of theorems of ZFC set theory hold in QST. Takeuti introduced predicates for commutativity and showed that appropriate modifications of ZFC axioms hold in his model. Since quantum logic does not ensure inferences from axioms to theorems in ZFC, a general transfer principle from ZFC theorems to valid statements in QST is desirable. In Ref. [2] such a transfer principle has been established for bounded ZFC theorems. Here, we push this result to a final form by establishing a general transfer principle for arbitrary ZFC theorems including unbounded ones to valid statements for models of QST based on an arbitrary complete orthomodular lattice with an arbitrary implication satisfying the requirement that its restriction to commuting statements reduces to the classical one. This new formulation of QST is general enough to incorporate with algebraic quantum field theory (AQFT). It is also discussed how this QST will provide a useful tool for the modal interpretation of AQFT introduced by Harvorson and Clifton [1].

[1] H. Halvorson and R. Clifton, *Maximal beable subalgebras of quantum mechanical observables*, ***Intnational Journal of Theoretical Physics***, vol. 38 (1999), pp. 2441–2484.

[2] M. Ozawa, *ransfer principle in quantum set theory*, ***The Journal of Symbolic Logic***, vol. 72 (2007), no. 2, pp. 625–648.

[3] G. Takeuti, *Quantum set theory*, ***Current issues in quantum logic*** (Beltrametti et al., editors), Plenum, London, 1981, pp. 303–322.

► MAKOTO TATSUTA, *Second-order system without implication and disjunction*.
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The type system with negation, product, and existential types is interesting since its type inhabitation is decidable [2], its type checking is undecidable, and it works as the target of many CPS translations [1]. This paper investigates the system NJ_2^- obtained from the type system by adding second-order universal quantification.

The system NJ_2^- is defined from the second-order natural deduction by removing implication and disjunction. Its formulas are defined by

$$A ::= X \mid \bot \mid \neg A \mid A \wedge A \mid \forall XA \mid \exists XA$$

where X is a propositional variable. Its inference rules are usual introduction/elimination rules for $\bot$, $\wedge$, $\forall$, $\exists$ as well as $\neg$ given as follows:

$$\dfrac{\begin{matrix}[A]\\ \vdots \\ \bot\end{matrix}}{\neg A}\ (\neg I) \qquad \dfrac{\neg A \quad A}{\bot}\ (\neg E)$$

We use Γ for a list of formulas. We say $A_1, \dots, A_n \vdash B$ is provable when B is provable under the assumptions $A_1, \ldots, A_n$. NK_2^- is defined as NJ_2^- plus classical logic.

THEOREM 1. *The provability in NJ_2^- is decidable. That is, there is an algorithm that decides the provability of any given $\Gamma \vdash A$.*

Glivenko's theorem holds for NJ_2^-.

THEOREM 2. *If $\Gamma \vdash A$ is provable in NK_2^-, then $\neg\neg\Gamma \vdash \neg\neg A$ is provable in NJ_2^-.*

Double Negation Shift holds in NJ_2^-.

THEOREM 3. $\forall X \neg\neg A \vdash \neg\neg\forall X A$.

We give a simpler proof of the following recent results.

THEOREM 4 (Sakagawa, Kashima). (1) $\neg\forall X A \vdash \exists X \neg A$.
(2) *If $\vdash A$ is provable in NK_2^-, then $\vdash A$ is provable in NJ_2^-.*

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[2] M. TATSUTA, K. FUJITA, R. HASEGAWA, AND H. NAKANO, *Inhabitance of existential types is decidable in negation-product fragment*, ***Proceedings of 2nd international Workshop on Classical Logic and Computation***, 2008.

► KEITA YOKOYAMA, *Formalizing non-standard arguments in second order arithmetic*.
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In Tanaka [2], we can find a model theoretic method to do non-standard analysis in WKL_0. By using this method, some popular arguments of non-standard analysis can be carried out in WKL_0. Similarly, we can use more techniques of non-standard analysis in ACA_0 and prove some theorems in ACA_0 [3]. Then, *can we canonically reconstruct formal proofs within* ACA_0 *or* WKL_0 *from such non-standard arguments?* This research is motivated by this question posed by Professor Sakae Fuchino. To formalize non-standard arguments, we introduce systems of non-standard second order arithmetic ns-ACA_0 and ns-WKL_0 corresponding to ACA_0 and WKL_0. In these systems, we can formalize the above non-standard arguments. By model constructions appearing in [2] and [3], we can show that ns-ACA_0 is a conservative extension of ACA_0 and ns-WKL_0 is a conservative extension of WKL_0, respectively. However, we need some canonical transformations that do not depend on semantics because to analyze non-standard techniques in second order arithmetic. To transform non-standard proofs directly into standard proofs, we interpret ns-ACA_0 in ACA_0 and interpret ns-WKL_0 in a conservative extension of WKL_0, as for the formalization of Harrington's conservation theorem by Avigad [1].

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[3] KEITA YOKOYAMA, *Non-standard analysis in* ACA_0 *and Riemann mapping theorem*, ***Mathematical Logic Quarterly***, vol. 53 (2007), no. 2, pp. 132–146.

► IKUO YONEDA, *CM-triviality in rosy theories*.
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CM-triviality is a geometric notion of non-forking independence relation introduced by

Hrushovski in [2]. I talk about CM-triviality in the thorn-forking and rosy theories' context. The usual definition for CM-triviality needs canonical bases of types: $\mathrm{Cb}(\bar{a}/A) \subseteq \mathrm{acl}^{\mathrm{eq}}(\mathrm{Cb}(\bar{a}/B))$ holds for any $\bar{a}, A, B \subseteq \mathcal{M}^{\mathrm{eq}}$ such that $\mathrm{acl}^{\mathrm{eq}}(\bar{a}, A) \cap B = A$ with $\mathrm{acl}^{\mathrm{eq}}(A) = A$ and $\mathrm{acl}^{\mathrm{eq}}(B) = B$. Since rosy theories do not necessarily have canonical bases with respect to the thorn-forking, we choose another definition: $\bar{a} \perp_A B$ implies $\bar{a} \perp_{A \cap \mathrm{acl}^{\mathrm{eq}}(\bar{a}, B)} B$ for any $\bar{a}, A, B \subset \mathcal{M}^{\mathrm{eq}}$ such that $A = \mathrm{acl}^{\mathrm{eq}}(A)$ and $B = \mathrm{acl}^{\mathrm{eq}}(B)$, where $\perp$ is a strictly independence relation by [1]. A rosy theory is a theory having a strict independence relation with local character.

We show that a rosy theory T is CM-trivial if and only if T has weak canonical bases of types with respect to the thorn-forking and $\mathrm{wcb}(\bar{a}/A) \subseteq \mathrm{acl}^{\mathrm{eq}}(\mathrm{wcb}(\bar{a}/B))$ holds for any $\bar{a}, A, B \subseteq \mathcal{M}^{\mathrm{eq}}$ such that $\mathrm{acl}^{\mathrm{eq}}(\bar{a}, A) \cap B = A$ with $\mathrm{acl}^{\mathrm{eq}}(A) = A$ and $\mathrm{acl}^{\mathrm{eq}}(B) = B$. We also show that CM-triviality is equivalent to the modularity in O-minimal theories with elimination of imaginaries. Unlike finite U-rank theories [4], let me note that neither local modularity nor CM-triviality are preserved under reducts in superrosy theories of finite $\mathrm{U}^{\mathrm{þ}}$-rank.

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[2] Ehud Hrushovski, *A new strongly minimal sets*, ***Annals of Pure and Applied Logic***, vol. 88 (1993), pp. 147–166.

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Abstracts of papers presented by title

► ANAHIT CHUBARYAN AND ARMINE CHUBARYAN, *Relative efficiency of Frege systems with different substitution rules*.

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We compare the proof complexities in Frege systems with multiple substitution rule and with constant bounded substitution rule. We use the generally accepted concept of Frege system $\mathcal{F}$ as follows: it uses a finite complete set of propositional connectives and it has a finite set of schematically defined rules of inference; furthermore it must be sound and complete.

A substitution Frege system $S\mathcal{F}$ consists of a Frege system $\mathcal{F}$ augmented with the substitution rule with inferences of the form $\frac{A}{A\sigma}$ for any substitution σ, consisting of a mapping from propositional variables to propositional formulas, and $A\sigma$ denotes the result of applying the substitution to formula A, which replaces each variable in A with its image under σ. This definition of substitution rule allows to use the simultaneous substitution of multiple formulas for multiple variables of A without any restrictions.

If for any constant integer $k \geqslant 1$ we allow substitution for only no more than k variables at a time, then we have k-bounded substitution rule. The k-bounded substitution Frege system

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$S_k\mathcal{F}$ consists of a Frege system $\mathcal{F}$ augmented with the k-bounded substitution rule.

We prove that

(1) for every fixed integers k_1 and k_2 and for every tautology φ, the minimal number of steps (the minimal size) of a $S_{k_1}\mathcal{F}$-proof of φ, and the minimal number of steps (the minimal size) of a $S_{k_2}\mathcal{F}$-proof of φ are polynomially related;
(2) for every fixed integer k and for every tautology φ, the minimal size of a $S_k\mathcal{F}$-proof of φ and the minimal size of a $S\mathcal{F}$-proof of φ are polynomially related;
(3) for every fixed integer k the minimal number of steps in a proof of a tautology in $S_k\mathcal{F}$ can be exponentially larger than in $S\mathcal{F}$.

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Let R be a commutative ring with identity. I will describe how the positive-primitive formulae in the language of left R-modules may be organized into a complex whose theory of homology coincides in dimension 0 with the K-theory of the free abelian category over R.

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In [4] it was proved that, *taken as systems of strict implication* in the sense of [3], the Lewis systems $\mathbf{S4}_+$ and **S4** are exactly contained as the *enthymematic implications* of Anderson–Belnap $\mathbf{E}_+$ and **E** of **entailment**. I show here that the same holds of Anderson–Belnap $\mathbf{T}_+$ and **T** of **ticket entailment** of [1]. The translations depend upon the definitions

D_+.	$A \supset B =_{df} A \wedge \mathbf{t} \to B$	(**S4+** strict implication)
D4.	$A \supseteq B =_{df} A \wedge \mathbf{t} \to B \vee \mathbf{f}$	(**S4** strict implication)
D□.	$\Box A =_{df} \mathbf{t} \to A$	(Necessity)

In these definitions, **t** is the *Ackermann constant* governed in **T** by the (2-sided) rule

$$\mathbf{Rt} \vdash A \text{ iff } \vdash \mathbf{t} \to A$$

and **f** is the constant governed by the definition

$$\mathbf{Df}\ \mathbf{f} =_{df} \sim \mathbf{t},$$

where $\sim$ is the *DeMorgan negation* originally supplied with **T**.

The result is somewhat surprising since **T**→ was originally advertised as *entailment shorn of modality* when introduced in [2]. The authors of [3] mentioned but declined to support my view that D□ is as appropriate for **T** as for **E**. I stand my ground!

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